Journal of Information & Communication Technology
Vol. 8, No. 2, (Fall 2014) 15-24

Efficent Design & Control of a Single link Iverted Pendulum using Fuzy logic Control

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ABSTRACT

Inverted pendulum is a highly non-linear system, which makes it a complex control problem that may be articulated for various real-life applications. Usually, its linearized model is implemented and controlled using a typical Proportional-Integral-Derivate (PID) controller but a different controller is required to provide more efficient control. The Fuzzy Logic controller provides a more smooth response. The controller is simulated on MATLAB/Simulink using PID, Fuzzy and hybrid control schemes.

Keywords : Inverted Pendulum, Fuzzy Logic, Fuzzy-PID, Comparison, MATLAB

* The material presented by the authors does not necessarily portray the viewpoint of the editors and the management of the Institute of Business and Technology (IBT) or Karachi Institute of Power Engineering.

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1 Introduction

Inverted pendulum control is a highly non-linear and unstable phenomenon that is a classical problem in control engineering. The unique nature and wide range of applications derived from this unstable system has invited interest from many researches and robotics enthusiasts. In fact, effective control of dynamically unstable and unbalanced mechanisms or systems such as an inverted pendulum is an ongoing and challenging area of research.

Generally, there are two types of controlling methodologies applied on inverted pendulum. First, where controller consists of a swing-up controller [1] and a stabilizing controller. Such controller, first swing up the pendulum from downward position to upwards and then stabilizes it there. It involves the design of both the controllers. As in [2] two swing up control scheme has been implemented that will switch to a stabilizing controller when the pendulum is within 5° degrees of the upright position. After that, the balance mode controller is used to stabilize and maintain the pendulum in the upright position. Second approach involves the design of a stabilizing controller only. In such controller design, the pendulum is physically held at some smaller angle initially. Due to hardware limitations, we are following the second approach in this project.

Propotional-Integral-Derivative (PID) controller is the conventional controller which is normally applied on the system, Fuzzy Logic controller (FLC) has also been applied on this system, such as in [1] a real-time control of the cart inverted pendulum system has been developed using Mamdani type inference system. Swing-up and stabilization of the inverted pendulum were implemented directly in a FLC. FLC delivers the best regulation and consumes relatively less control energy as compared to the conventional control schemes [3].

In this paper, we will explore different types of controllers using Fuzzy Logic and hybrid techniques and use PID controller output as a reference for comparison. The simulation of these controllers on the system is performed in MATLAB/ Simulink environment. The results verify the performances of the controllers in achieving vertical balance of the pendulum as well as provide a comparison of these controllers in terms of response to external disturbances and net control energy.

2 Mathematical Modeling

The mathematical modeling of the inverted pendulum includes simple calculation of forces on the system, in both the horizontal and vertical direction. The model is to be used to find the transfer function of the system.

Horizontal direction Forces: In Fig.1 (b) the sum of forces on the cart is: $M\ddot{x} + b\dot{x} + N = F$

Force due to moment on pendulum: $\tau = r \times F = I\ddot{\theta}$

And $$F = \frac{I\ddot{\theta}}{r}$$

Put $r = l$ and $I = ml^2$: $$F = \frac{ml^2}{l}\ddot{\theta} = ml\ddot{\theta}$$

Force Component of Centripetal Force (in horizontal direction):

$$F = \frac{I\dot{\theta}^2}{r} = \frac{ml^2\dot{\theta}^2}{l}$$

From Fig.1 (c): $N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta$

And as $$M\ddot{x} + b\dot{x} + N = F$$

$$(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \qquad (1)$$

Forces in Perpendicular direction in Fig.1(c):

$$P\sin\theta + N\cos\theta - mg\sin\theta = ml\ddot{\theta} + m\ddot{x}\cos\theta \qquad (2)$$

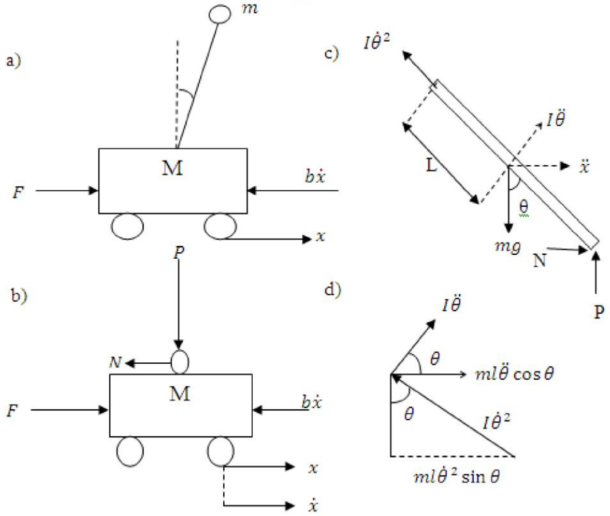

Fig.1. a) A single link inverted pendulum, b) Free-body diagram c) Forces acting on the pendulum, d) Forces around the centroid of pendulum.

Sum of forces around centroid of Pendulum in Fig.1 (d):

$$-Pl\sin\theta - Nl\cos\theta = I\ddot{\theta}$$

$$P\sin\theta + N\cos\theta = -\frac{I\ddot{\theta}}{l}$$

Put in equation (2): we get $-\frac{I\ddot{\theta}}{l} - mg\sin\theta = ml\ddot{\theta} + m\ddot{x}\cos\theta$

$$(I + ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\text{co} \qquad (3)$$

If we take frictional losses negligible, we take b=0:
Equation (1) becomes:

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \quad (4)$$

From equation (3):

$$\ddot{x} = \frac{(I+ml^2)\ddot{\theta} + mgl\sin\theta}{-ml\cos\theta}$$

And from equation (4):

$$\ddot{x} = \frac{F+ml\dot{\theta}^2\sin\theta - ml\ddot{\theta}\cos\theta}{(M+m)} \quad (5)$$

Equating both:

$$(I+ml^2)\ddot{\theta} + mgl\sin\theta = -ml\cos\theta\left[\frac{F+ml\dot{\theta}^2\sin\theta - ml\ddot{\theta}\cos\theta}{(M+m)}\right]$$

Put $I = \frac{ml^2}{3}$

$$\left(\frac{4}{3}ml^2\right)\ddot{\theta} + mgl\sin\theta = \frac{-ml\cos\theta F}{(M+m)} - \frac{m^2l^2\dot{\theta}^2\sin\theta\cos\theta}{(M+m)} + \frac{m^2l^2\ddot{\theta}\cos^2\theta}{(M+m)}$$

$$\ddot{\theta}ml^2\left[\frac{4}{3} - \frac{m\cos^2\theta}{(M+m)}\right] = -mgl\sin\theta + ml\cos\theta\left[-\frac{F}{(M+m)} - \frac{ml\dot{\theta}^2\sin\theta}{(M+m)}\right]$$

$$\ddot{\theta} = \frac{-g\sin\theta + \cos\theta\left[\frac{-F-ml\dot{\theta}^2\sin\theta}{(M+m)}\right]}{l\left[\frac{4}{3} - \frac{m\cos^2\theta}{(M+m)}\right]} \quad (6)$$

Equation (5) and (6) represent the non-linear model of the inverted pendulum.

Linear Approximation:

Now, if we approximate angle to near vertical position. And if angle is small, its rate, and its square can be neglected, also and the relations become:

$$\ddot{x} = \frac{F-ml\ddot{\theta}}{(M+m)} \quad (7)$$

$$\ddot{\theta} = \frac{-g\theta + \left(\frac{-F}{M+m}\right)}{l\left[\frac{4}{3} - \frac{m}{(M+m)}\right]} \quad (8)$$

2.2 System Parameters and Open Loop System Analysis

The system parameters used for simulation are shown in Table 1.

Table 1: System parameters used for simulation

Parameters	Value	Units
Cart Mass 'M'	1.4	Kg
Pendulum Mass 'm'	0.5	Kg
Pendulum half length 'l'	0.5	Meter
Friction coefficient 'b'	0	--
Gravitational Force 'g'	9.8	m/s^2

The open loop system response is highly unstable (Fig.2), with sudden overshoots of both the angle and displacement from zero initial conditions.

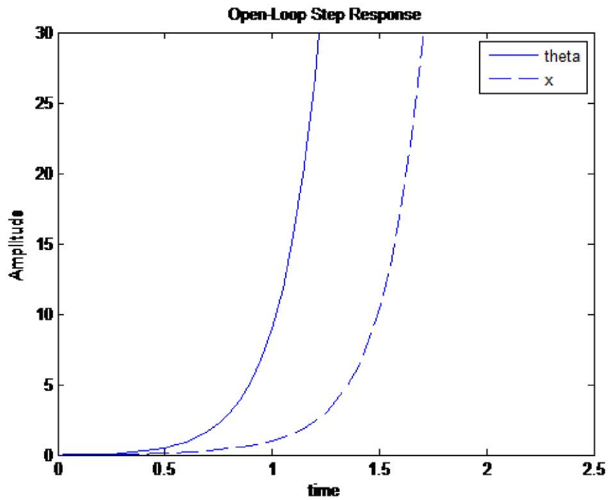

Fig.2. Open Loop response of the system.

3 Controller design

3.1 PID controller

The proportional-integral-derivative (PID) controller is an extensively used conventional controller scheme. The Proportional-Integral-Derivative controller has the following form:

$$C(s) = K_p + \frac{K_i}{s} + K_d s$$

This controller works on the error in the process variable. The controller parameters to be tuned are the gains, K_p (proportional gain), K_i (Integral gain) and K_d (Derivative gain). Proportional controller makes the step-response faster, derivative control improves the transient response of the system and Integral controller removes the steady state error. The controller is inserted in feedback as follows:

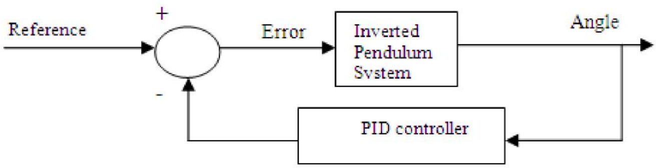

Fig.3. PID controller applied on the system.

An educated guess of the gains of PID controller with sufficient number of trials, yields: K_p=80, K_i=40 and K_d=40

3.2 Fuzzy Logic Controller (FLC)

Fuzzy logic is based on fuzzy sets, which represent uncertainty. Fuzzy control provides a formal methodology for implementing a human's heuristic knowledge about how to control a system.

The fuzzy controller has four main components [4-6]: The rule base, the inference mechanism, the fuzzification interface and the defuzzification interface. The Fuzzy Logic controller developed for the linear model is a Mamdani type controller, whose inputs and output have three membership functions each, namely N (Negative), Z (Zero) and P (Positive).

a)
b)
c)

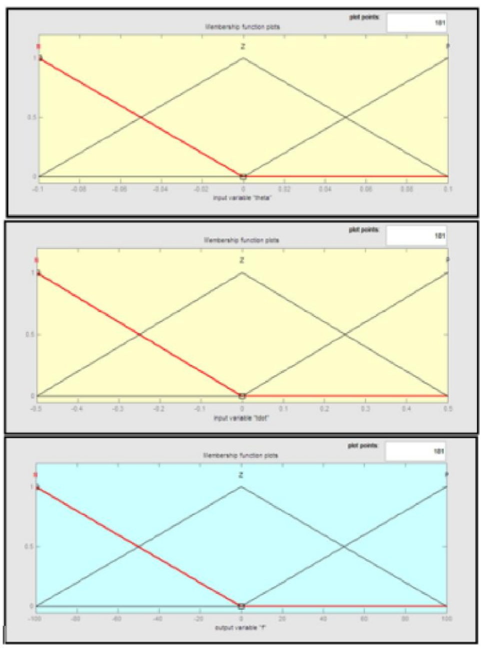

Fig.4. Membership functions of the Inputs a) theta b) theta-dot and Output of FLC; c) Force

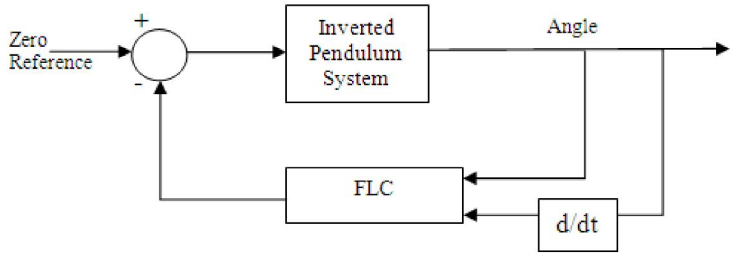

Fig.5. Fuzzy Logic Controller applied on the system

Figure 5 shows the FLC controller applied on the system. The rules or the rule-base of the fuzzy logic are based on if-else conditions. For example,

If **theta** is N (Negative) and **theta-dot** is Z (Zero), then **Force** is Z (Zero).

Nine such rules make up the rule base as shown in Table 2.

Table 2: 3x3 rule base for fuzzy logic controller

Theta \ Theta dot	N	Z	P
N	N	N	Z
Z	N	Z	P
P	Z	P	P

3.3 Hybrid of PID and Fuzzy Controller

In hybrid controller design, the controller gains of PID controllers namely Kp, Ki and Kd are tuned by a Fuzzy Logic Tuner. Such type of controller was designed on a hydraulic actuator in [7]. The scheme of this controller is shown in Figure 5. The significance of this type of tuning lies in the fact that, the behaviour of PID controller gains is controlled by the Fuzzy inference method.

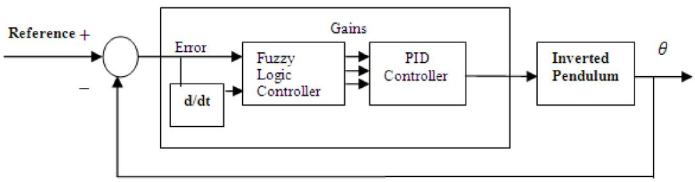

Fig.6. Fuzzy-PID controller scheme

When the system is running in real-time, the output of the FLC constantly changes thus giving variable gains for the PID controller. In essence, this controller is a cascade of the PID and FLC. So the PID action is applied on the Fuzzy output which is achieved through the rule base designed to act on the error in the angle. The Fuzzy controller takes inputs: error 'e' and rate of error 'de'. Five membership functions of the inputs are built; these are the same as used in Fuzzy-PID controller. Because both are working on same set of inputs, so the universe of discourse and the membership functions remain same, which give the required performance. The rule base is shown in Table 3.

Table 3: 5x5 rule base for FLC for non-linear model

e \ de	NL	NS	Z	PS	PL
NL	PL	PL	PL	PS	Z
NS	PL	PL	PS	Z	NS
Z	PL	PS	Z	NS	NL
PS	PS	Z	NS	NL	NL
PL	Z	NS	NL	NL	NL

4 Simulation results

a) Angle

The initial condition of the angle is set as 0.1 radians, which is equal to 5.73°. In ideal condition, the angle should go instantly to zero value or the pendulum should approach the vertical position.

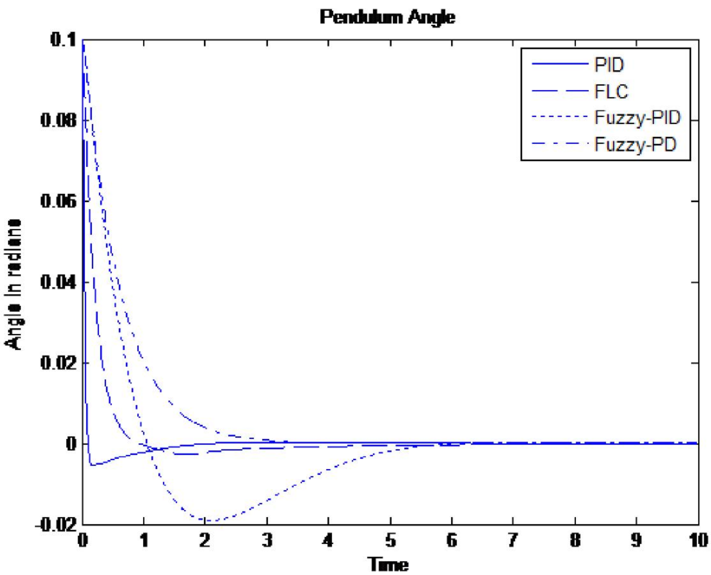

Fig.7. Angle of the inverted pendulum with initial condition 0.1 radians.

The results of the angle from all controller in Fig.6 shows that PID controller makes an abrupt change in angle whereas the Fuzzy Logic controller makes it smoother. The Fuzzy-PID tuner gives the slowest response. But still regulates the angle in 5.5 seconds.

b) Control Force of Controllers

To compare the performances of the controllers, we need a plot of the control force generated, which is in Fig.7.

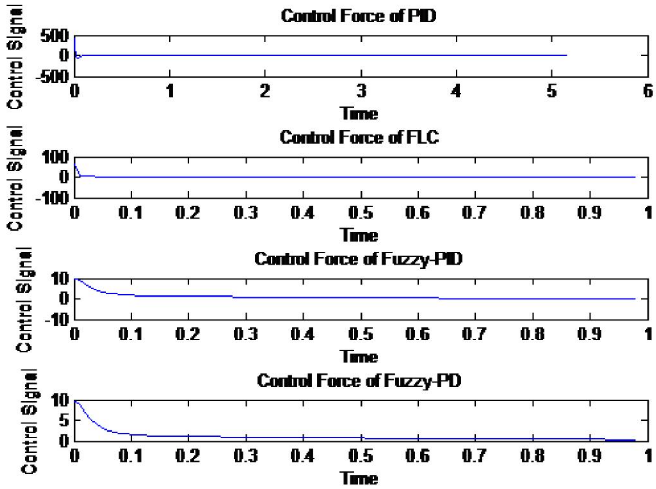

Fig.8. The signal which goes to the system from controller

Here initial force of PID is 408N, initial force of FLC is 67.3N and initial force of Fuzzy-PID and Fuzzy-PD is 9.677N. Overall analysis shows that the Fuzzy-PID controller is a smooth and energy efficient controller. The smoothness is evident from the angle response, which is the slowest because of the lowest energy used. The cart travels a bit far in the beginning but remains steady after that.

$$Energy = \int F(t)^2 dt$$

5 Performance Comparison

The performance comparison of the controller is based on the net control energy they provide to the system. The formula to calculate energy of a signal is as follows [3]:

We calculated the control energy of all the controllers, as in Table 4.

Table 4: Performance Comparison of controllers

Controller	Net Control Energy
PID	1.1436×10^5
FLC	1.6392×10^3
Fuzzy-PID	363.2849
Fuzzy-PD	337.086

The PID controller takes the highest control energy among the controller schemes applied, Fuzzy Logic controller takes a fraction of that and

Fuzzy-PID controllers and the non-linear Fuzzy logic controller take the least control energy. This comparison verifies the reason, why the Fuzzy Logic controller is chosen for this control design requirement. The Fuzzy-PID controller has improved performance over the simple PID controller, which was also verified in [8].

6 Conclusion

The FLC gives appropriate angle response with very less force applied to the cart. The task of controlling the unstable system of inverted pendulum seems very easy with the use of FLC. It's because of the use of if-else condition, and the fact that FLC does not depend on the transfer function of the system. The controller response to the inputs just depends on the universe of discourse and the rule bases. So, this controller is an adequate choice for control of such non-linear systems.

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